OCR Mechanics 4 January 2002

1

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (equivalent of the linear $s = ut + \frac{1}{2})at^2$)

$96 = 5 \times 24 + \frac{1}{2}\alpha \times 25$

$\alpha = {}^{-}$ rad

so the angular decelleration is $\mathbf{1 \cdot 92}$ **rad s⁻²** (show)

[2]

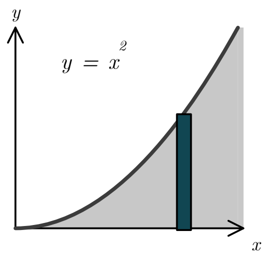
$\omega_1{}^2 = \omega_0{}^2 + 2\alpha\theta$ ("$v^2 = u^2 + 2as$")

$0 = 24^2 + 2 \times {}^{-}1 \cdot 92 \times \theta$

$\boldsymbol{\theta} = \mathbf{150}$ **rad**

[2]

2



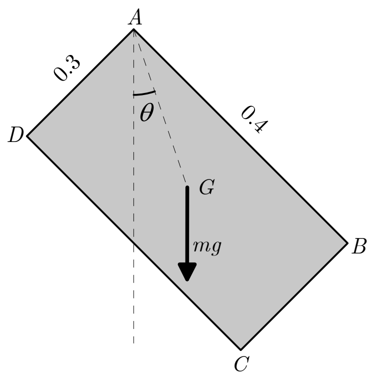
mass of 'elemental disc' $= \rho.\pi y^2.\delta x = 5400\pi x^4 \delta x$

MoI of 'elemental disc' $= \frac{1}{2}mr^2 = 2700\pi x^8 \delta x$

$I = \int_0^1 \left(2700\pi x^8\right) \mathrm{d}x = 300\pi = \mathbf{942}$ **kg m²** (3 s.f.)

[5]

3



$I_{AB} = \frac{4}{3}ml^2 = \frac{4}{3} \times 0 \cdot 6 \times 0 \cdot 15^2 = 0 \cdot 018$

$I_{AD} = \frac{4}{3}ml^2 = \frac{4}{3} \times 0 \cdot 6 \times 0 \cdot 20^2 = 0 \cdot 032$

perpendicular axes rule ….

$I_A = 0 \cdot 018 + 0 \cdot 032 = \mathbf{0 \cdot 05}$ **kg m²**

[3]

$AG = 0 \cdot 25$ (Pythagoras)

when AG makes angle θ with the vertical ...

$$I\ddot{\theta} = {}^{-}mg(0 \cdot 25 \sin\theta)$$

$$\ddot{\theta} = {}^{-}29 \cdot 4 \sin\theta$$

and so for small θ ...

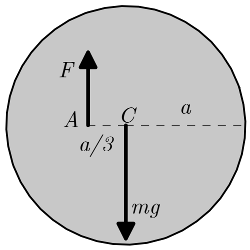
$$\ddot{\theta} \approx {}^{-}29 \cdot 4\theta$$

so the motion is approximately SHM with

$$T = \frac{2\pi}{\sqrt{29 \cdot 4}} = 1 \cdot 15879... = \mathbf{1 \cdot 16 \text{ s}} \quad \text{(3 s.f.)}$$

[3]

4



parallel axes rule …

$$I_A = I_C + m\left(\tfrac{1}{3}a\right)^2 = \tfrac{1}{2}ma^2 + \tfrac{1}{9}ma^2 = \boldsymbol{\tfrac{11}{18}ma^2}$$

[2]

on release …

$$\mathrm{M}(A) \qquad C = I\ddot{\theta}$$

$$mg\left(\tfrac{1}{3}a\right) = \left(\tfrac{11}{18}ma^2\right)\ddot{\theta}$$

$$\boldsymbol{\ddot{\theta} = \tfrac{6g}{11a}}$$

[2]

When released from rest no 'central' force is needed to maintain circular motion about A so **the force on the disc a A is purely vertical (upwards)**.

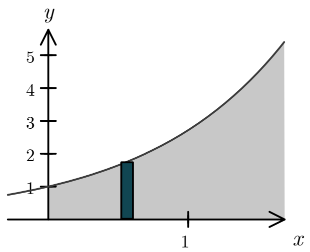
[1]

$$\mathrm{N2}(\downarrow) \qquad mg - F = m\left(r\ddot{\theta}\right)$$

$$F = mg - m\left(\tfrac{1}{3}a\right)\left(\tfrac{6g}{11a}\right) = \boldsymbol{\tfrac{9}{11}mg}$$

[3]

5



$$m = \rho\int_0^{\ln 5} e^x \,\mathrm{d}x = 4\rho$$

$$4\rho\,\overline{x} = \int_0^{\ln 5} \rho\, xe^x \,\mathrm{d}x = \rho\left[xe^x\right]_0^{\ln 5} - \rho\int_0^{\ln 5} e^x \mathrm{d}x = \rho.5\ln 5 - 4\rho$$

$$\boldsymbol{\overline{x} = \tfrac{5}{4}\ln 5 - 4} \qquad \text{(show)}$$

[5]

using the same 'strips' …

$$4\rho\overline{y} = \int_0^{\ln 5} \left(\tfrac{1}{2}y\right).\rho y \,\mathrm{d}x = \int_0^{\ln 5} \tfrac{1}{2}\rho e^{2x} \mathrm{d}x = \rho\left[\tfrac{1}{4}e^{2x}\right]_0^{\ln 5} = \tfrac{1}{4}\rho(25-1) = 6\rho$$

$$\boldsymbol{\overline{y} = 1\cdot 5}$$

[3]

6

$$I = 124\times 6^2 + \left(\tfrac{1}{3}\times 75\times 3\cdot 6^2 \;+\; 75\times 2\cdot 4^2\right) = 4464 + 756 = \mathbf{5220\ \ kg\,m^2} \qquad \text{(show)}$$

[3]

energy considerations …

gain in K.E. = loss in G.P.E. − work done by frictional couple

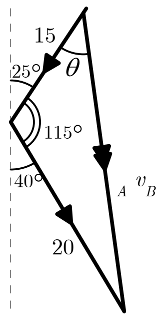
$$\tfrac{1}{2}I\omega^2 = mgh - C\theta$$

$$2610\omega^2 = 75\times 9\cdot 8\times 2\cdot 4 - 850\times \tfrac{\pi}{2}$$

$$\boldsymbol{\omega = 1\cdot 72\ \ \mathbf{rad\,s^{-1}}}$$

[5]

7 ${}_A\mathbf{v}_B = {}_A\mathbf{v}_G - {}_B\mathbf{v}_G$

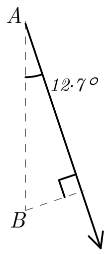


$$\left|{}_A\mathbf{v}_B\right|^2 = 20^2 + 15^2 - 2\times 20\times 15\times\cos 115^\circ = 878\cdot 640...$$
$$\left|{}_A\mathbf{v}_B\right| = 29\cdot 6406... = \mathbf{29\cdot 6\ \ km\,h^{-1}} \qquad \text{(3 s.f.)}$$

$$\frac{\sin\theta}{20} = \frac{\sin 115^\circ}{29\cdot 640...}$$
$$\theta = 37\cdot 7^\circ$$

the relative velocity is on **bearing 167°** [4]

now considering the **position** of A relative to B …..



closest distance $= 70\sin 12\cdot 7^\circ = \mathbf{15\cdot 4\ \ km}$

$$t = \frac{70\cos 12\cdot 7^\circ}{29\cdot 64} = 2\cdot 30\,\text{hrs}$$

ships closest together at 2:18 am

[5]

8 relative to $\theta = 0$ position …

$$V = {}^{-}mg\left(a\sin\theta\right) + \tfrac{1}{2}\times\frac{\tfrac{1}{2}mg}{a}\left(2a\sin\theta\right)^2 = \boldsymbol{mga}\left(\mathbf{sin^2}\boldsymbol{\theta} - \mathbf{sin}\,\boldsymbol{\theta}\right) \qquad \text{(show)}$$

[3]

$$\frac{dV}{d\theta} = mga\left(2\sin\theta\cos\theta - \cos\theta\right) = mga\cos\theta\left(2\sin\theta - 1\right)$$

so for $0 < \theta < \pi$ the system is in equilibrium for $\boldsymbol{\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$ [5]

$$\frac{d^2V}{d\theta^2} = mga\left(\sin\theta - 2\sin^2\theta + 2\cos^2\theta\right)$$

$\theta = \frac{\pi}{6}$ $\quad \frac{d^2V}{d\theta^2} = \frac{3}{2}mga > 0$ $\quad \therefore$ **stable**

$\theta = \frac{\pi}{2}$ $\quad \frac{d^2V}{d\theta^2} = {}^{-}mga < 0$ $\quad \therefore$ **unstable**

$\theta = \frac{5\pi}{6}$ $\quad \frac{d^2V}{d\theta^2} = \frac{3}{2}mga > 0$ $\quad \therefore$ **stable**

[4]